If you have been following my last few posts, I am sure you are a little wary of today's post. They have been a little convoluted lately since we are dealing with permutations and combinations. Next, we will tackle probability but today, I am going to digress (to give you some much needed respite) and take up a simple yet interesting topic. We deal with quadratic equations on a regular basis. Tackling them effectively is pretty much one of the most basic and important skills you need for GMAT Quant. Today we will look at some relationships between the coefficients of quadratic equations and roots.

I am sure you know how to solve a quadratic equation so I will not delve into that. I am also assuming that you studied the Vieta's formulas in high school.

Let me recap it here:

Given a quadratic equation $ax^2 + bx + c = 0$ with roots p and q,

Sum of the roots = $p + q = -b/a$

Product of the roots = $pq = c/a$

It is good to remember these relations since they can be useful sometimes. (You can easily derive them by expanding the following: $ax^2 + bx + c = a(x – p)(x – q)$)

Let us go through a quick example.

Example: Two friends, Ann and Beth started solving a quadratic equation. Ann made a mistake while copying the constant term and got the roots as 5 and 9. Beth made a mistake in the coefficient of x and she got the roots as 12 and 4.What is the equation?

(A) $x^2 + 4x +14 =0$

(B) $2x^2 +7x -24 =0$

(C) $x^2 -14x +48 =0$

(D) $3x^2 -17x +52 =0$

(E) $2x^2 + 4x +14 =0$

Solution:

Ann made a mistake while copying the constant term i.e. c but she copied a and b properly. So the sum of the roots she found must be correct.

$-b/a = 5 + 9 = 14$

Beth made a mistake while copying the coefficient of x i.e. b but she copied a and c properly. So the product of the roots she found must be correct.

$c/a = 12*4 = 48$

Only option (C) above has $–b/a = 14$ and $c/a = 48$. Therefore, answer must be (C).

You see knowing the Vieta's formulas made a potentially tricky question pretty simple.

Now on to the real thing I had in mind for this post. Once, someone asked me the following question: Is there a quick way to know whether a quadratic equation has one or two positive solutions without the need to solve it?

It made me think how we know so much but are still sometimes unable to connect the dots between inter-linked concepts. The following was my answer to the query. I hope you find it useful too.

Answer: Yes, there is. The Vieta's formulas help us figure it out.

Let's say we have a quadratic equation: $ax^2 + bx + c = 0$

We know that sum of the roots = $-b/a$

Product of the roots = $c/a$

(We are dealing with only real roots.)

Case 1: Sum of the roots is positive; product of the roots is positive.

If product is positive, it means both roots are either positive or both are negative. Since the sum is also positive, both roots must be positive.

For example: $2x^2 – 8x + 1 = 0$

Product of roots = 1/2

Sum of roots = $-(-8)/2 = 4$

Both roots positive.

Case 2: Sum of the roots is negative; product of the roots is positive.

If product is positive, it means both roots are either positive or both are negative. Since the sum is negative, both roots must be negative.

For example: x^2 + 5x + 4 = 0
Product of roots = 4/1 = 4
Sum of the roots = -5/1 = -5
Both roots negative.
Case 3: Product of the roots is negative.
If product is negative, it means one root is positive and the other is negative. If the sum is positive, the absolute value of the positive root is higher than the absolute value of the negative root. If the sum is negative, the absolute value of the negative root is higher than the absolute value of the positive root.
For example: 3x^2 -6x -2 = 0
Product of roots = -2/3
One root negative, one positive.
These little insights help you solve quirky questions quickly and neatly. Be on the lookout for inferences you can draw from the knowledge you already possess and as always, keep practicing!